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Understanding convexity

***Convexity is a mathematical concept** used to compare a bond's upside price potential with its downside risk. A bond has positive convexity when the price increase resulting from a given decline in interest rates is greater than the price decrease that would result from an equivalent rise in rates. As a general rule, noncallable bonds have positive convexity, while many bonds that can be redeemed prior to maturity have negative convexity.*

WHY DO WE START WITH DURATION?

To understand convexity, we must first understand duration, which is a metric used to estimate how much the price of a bond will change in response to a change in its yield. Modified duration is a particular version of duration, which, when multiplied by the change in yield, approximates the expected percentage change in price.

However, using duration to estimate price changes assumes a linear relationship between the change in yield and the change in price. In other words, the percentage change in price from an increase in yield is presumed to be the same as the change in price from a decrease in yield.

Price estimates based on duration alone plot in a straight line, while convexity draws a curved line that turns up from the line based on duration (Figure 1).

Figure 1: Convexity measures the curvature of the relationship between bond yields and prices

Bond price

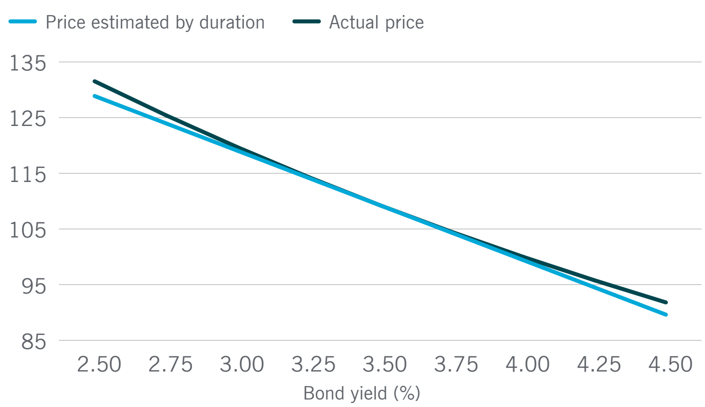

Chart calculates the yield and price of a 30-year noncallable bond. These examples are hypothetical and in no way intended to represent the performance of any Nuveen investment.

For example, a 10-year bond with a 4.00% coupon and a 3.50% yield would have a price of 104.19 and a modified duration of 8.24 years. If interest rates rise 1.00%, the price of the bond would be expected to fall by 8.24% (1.00% times 8.24 equals 8.24%) to an estimated value of 95.61. Likewise, if interest rates fall by 1.00%, the price of the bond would be expected to rise by 8.24% to 112.77 (Figure 2).

However, the actual prices in each case would be slightly higher. The difference between the actual price and the predicted price based on duration is a function of the bond's convexity.

Figure 2: Using modified duration to estimate bond prices

Assume a 10-year bond with a 4.00% coupon issued at a 3.50% yield

Yield change: 1.00%

Coupon	Yield	Modified duration	Estimated price from duration	Actual price
4.00%	3.50%	8.24		104.19
4.00%	4.50%	8.12	95.61	96.01
4.00%	2.50%	8.35	112.77	113.20

Notice that as the yield increases from 3.50% to 4.50%, the duration shortens from 8.24 to 8.12, while a decline in yield lengthens duration from 8.24 to 8.35. In other words, the price becomes more volatile when moving higher, and less volatile when moving lower. Thus, the bond has more upside than downside potential for the same change in interest rates, which is precisely what convexity quantifies.

HOW DOES CONVEXITY RELATE TO DURATION?

Duration estimates the rate of change in price for a given change in yield, while convexity corresponds to the rate of change in duration (i.e., a rate of change in a rate of change). Using duration is like estimating the distance a car will travel if going at a constant speed; using convexity is like estimating the distance traveled by a car that is accelerating.

We can estimate the convexity expressed as a percentage change in price using the following formula to compute how much the increase in price exceeds the decrease in price:

$$\frac{\text{RisingPrice} + \text{FallingPrice} - (2 * \text{StartingPrice})}{\text{StartingPrice} * 2}$$

In our example, the formula becomes:

$$\frac{113.20 + 96.01 - (2 * 104.19)}{104.19 * 2}$$

$$= \frac{209.21 - 208.38}{208.38}$$

$$= \frac{0.8321}{208.38}$$

$$= 0.3993\%$$

Multiplying 0.3993% by the dollar price of 104.19 gives a change in price of 0.42 due to convexity. Since we estimated convexity based on a 1.00% change in yield, and since duration provides an estimate of the percentage change in price resulting from a 1.00% change in yield, we can add the changes in price due to duration and convexity to estimate the bond's change in price more closely.

Figure 3: Combining price estimates from duration and convexity

Estimated price from duration	Price change due to convexity	Estimated price from duration and convexity	Actual price
95.61 +	0.42 =	96.03	96.01
112.77 +	0.42 =	113.19	113.20

WHAT ABOUT CALLABLE BONDS?

For noncallable bonds, which cannot be redeemed prior to their stated maturity, using modified duration for our calculations was sufficient. However, callable bonds require using effective duration, which weighs the probability of a bond being called.

Many callable bonds have negative convexity, especially those priced near par. Their upside potential is limited by the issuer's ability to redeem the bonds and refinance at a lower interest rate.

In our example, if the 10-year bond with a 4.00% coupon could be called in five years, it would initially be priced to the call date at 102.28. An increase in yield from 3.50% to 4.50% would cause the value to drop by -6.13% to 96.01, as the bond would then be priced to maturity. A decline in yield to 2.50% would only increase the price by 4.63% to 107.01.

$$\frac{107.01 + 96.01 - (2 * 102.28)}{102.28 * 2}$$

$$= \frac{203.02 - 204.56}{204.56}$$

$$= \frac{-1.532}{204.56}$$

$$= -0.75\%$$

In the municipal bond market, many bonds were originally issued as premium bonds with high coupon rates, which reduces the likelihood that interest rates would rise to the point that the bonds would no longer be called.

At the end of September 2023, 60% of the market value of the Standard & Poor's Municipal Bond Index consisted of bonds with coupon rates between 5.00% and 5.50%. From 31 Aug 2022 to 29 Sep 2023, the average yield of the index climbed from 3.38% to 4.44%, and the average dollar price fell by more than $4.50.

The recent increase in interest rates has decreased the probability that the bonds would be called, thereby lengthening the bonds' effective duration. In addition, the convexity of many bonds became negative as the prices of bonds dropped from well above par to close to par.

MEASURES OF CONVEXITY REFINE THE CALCULATION

While duration is useful for estimating how volatile the price of a bond will be in response to changing interest rates, measures of convexity refine the calculation to provide a closer estimate to the actual price. In the case of callable bonds, convexity can indicate the extent to which the call option disadvantages the bondholder.

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While duration is useful *for estimating how volatile the price of a bond will be in response to changing interest rates,* ***convexity refines the calculation.***

OPINION PIECE. PLEASE SEE IMPORTANT DISCLOSURES IN THE ENDNOTES.

APPENDIX

WHAT IF YIELDS CHANGE MORE/LESS THAN 1.00%?

In our example, we shifted yields by 1.00% to estimate convexity. This resulted in a number comparable to duration, a measure that also predicts how prices would change if yields shift by 1.00%.

Convexity can also be estimated using smaller yield changes. As we noted, a 10-year bond with a 4.00% coupon and a 3.50% yield would have a price of 104.19 and a modified duration of 8.24 years. If interest rates rise by 0.25%, the price of the bond would be expected to fall by 2.06% (0.25% times 8.24 equals 2.06%). The estimated value of the bond would decline by 1.0206 times 104.19, or 102.04. If interest rates fell by 0.25%, the value would increase by 2.06% to 106.33.

Figure 4: Calculating convexity using smaller yield changes

Assume a 10-year bond with a 4.00% coupon issued at a 3.50% yield

Yield change: 0.25%

Coupon	Yield	Modified duration	Estimated price	Actual price
4.00%	3.50%	8.24		104.19
4.00%	3.75%	8.21	102.04	102.07
4.00%	3.25%	8.27	106.33	106.36

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Convexity can also be estimated *using smaller yield changes.*

We can estimate the convexity expressed as a percentage change in price by using the formula we used for a 1.00% change in yield:

$$\frac{106.36 + 102.07 - (2 * 104.19)}{104.19 * 2}$$

$$= \frac{208.43 - 208.38}{208.38}$$

$$= \frac{0.05}{208.38}$$

$$= 0.024\%$$

To express convexity so that it is consistent with the way duration is expressed based on a 1.00% change in yield, we must modify the formula to include a factor that will adjust our estimate to reflect the change in price due to a 1% change in yield. In the denominator, we include the square of the ratio between the number of basis points by which we changed the yield and 100 or (BPShift/100)^2.

$$\frac{\text{RisingPrice} + \text{FallingPrice} - (2 * \text{StartingPrice})}{\text{StartingPrice} * (\text{BPShift}/100)\hat{}2 * 2}$$

In our example,

$$\frac{106.36 + 102.07 - (2 * 104.19)}{104.19 * (25/100)\hat{}2 * 2}$$

$$= \frac{208.43 - 208.38}{104.19 * 0.0625 * 2}$$

$$= \frac{0.052}{13.02}$$

$$= 0.3990\%$$

If we start with convexity and want to estimate the change in price for a change in yield other than 1.00%, we must first multiply the convexity by (BPShift/100)^2, which in the case of a shift of 25 basis points (0.25%) would be 0.0625 x 0.3990% = 0.025%. This is roughly what we found in our original estimate based on changing yields by 0.25%. (The ratio of 1.00% to 0.25% is 4; the inverse of 0.0625 is 16 or 4^2. The ratio of 0.3990 to 0.025 is also 16.)

For more information, please visit nuveen.com.

Endnotes

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